(Online) ISSN 2712-0554
Original Research

Heritage and Sustainable Development
Vol. 7, No. 2, 2025, pp.771-782
https://doi.org/10.37868/hsd.v7i2.1386

A 5D chaotic macroeconomic model for simulating policy synchronization in developing economies

Abdulsattar Abdullah Hamad[1*], P. Murugabharathi[2], Hameed Hasan Khalaf[3],
[1] Department of Physics, College of Education, University of Samarra, Salah Al-Din, Iraq
[2] Bharath Institute of Higher Education and Research, Selaiyur, India
[3] Tikrit University, College of Economic and Administration, Iraq

*Corresponding author E-mail: a.a.hamad@uosamarra.edu.iq

Received May 09, 2025
Revised Jul. 2, 2025
Accepted Jul. 8, 2025
Online Jul. 21, 2025

Abstract

This paper proposes a novel approach that lies in the structure of the equations and the economic application to reflect complex nonlinear interactions between key macroeconomic variables such as inflation, exchange rate, unemployment, GDP, and foreign investment. The model contains nonlinear differential equations coupled with feedback terms that induce hypersonic behavior. This structure enables the study of economic instability under various parameter settings and analysis of control strategies stabilizing the economy. Numerical simulation displays the sensitivity of the system towards initial conditions and the presence of many positive Lyapunov exponents. The proposed model provides insight into economic fragility and allows the design of strong policy interventions.

Keywords: 5D chaotic system, Macroeconomic modeling, Inflation dynamics, Hyperchaos, Nonlinear control, Economic synchronization

1. Introduction

Economic systems, especially in developing countries, are clearly connected and are highly sensitive to both internal and external disturbances. Traditional linear models often fail to accurately hold the underlying instability of developing economies [1-2]. As global financial structures become more complex, economic activities are rapidly connected, and political instability increases, a pressure is required for an advanced modeling structure. Drawing on the principles of chaotic dynamic systems, this study introduces a non-five-dimensional model capable of capturing the complex and chaotic behavior of economic fluctuations. The model aims to assist policymakers in identifying systemic risks, predicting crises, and implementing coordinated regulatory strategies.

Economic systems in developing countries are particularly prone to instability and sudden ups and downs, reflecting a high level of sensitivity to instability. These unexpected changes create important challenges for economists and policymakers trying to achieve economic stability and sustainable development [3]. While traditional linear economic models serve as a useful tool for basic forecasting, they are often less complex, dynamic, and low in capturing non-cylinder interactions that are characteristic of real-world economies. Such models ignore important aspects, including interlinking economic relations, response systems, delays of time, and structural imbalance [4-6]. Today's global economy is rapidly taking shape through interconnected forces - from capital flow and currency systems to trade integration and geopolitical changes. As a result, even local economic disruption can increase widespread crises through complex systemic linkage [7-9]. These realities call for more advanced modeling approaches that are capable of embracing non-surprise, reaction dynamics, and sensitive dependence on early conditions, which define the characteristics of ALIC.

Inspired by the field of dynamical systems theory, this study proposes a novel five-dimensional (5D) nonlinear chaotic model to analyze macroeconomic instability in developing economies. The model incorporates key economic indicators and interaction parameters, enabling realistic simulations of chaotic behavior in economic variables [10-14].

The proposed model is designed to reflect complex economic mobility seen in unstable areas by incorporating major macroeconomic indicators such as inflation, exchange rates, unemployment, GDP and foreign direct investment. Instead of mimicking cyclic economic fluctuations only, the model wants to examine the root causes of instability, mimics the possible consequences of various policy interventions, and evaluate the control mechanism through synchronization-based analysis [15]. In this context, the study implements the concept of projective synchronization, which is derived from anarchic theory, how to find out how non-inviting strategies can be employed to stabilize or coordinate economic systems under unstable conditions [16]. This is particularly important in environment where direct intervention does not give linear or proportional results. The degree of coupling between projective synchronization systems allows for individual degrees, which can represent individual intensity or asymmetric responses in economic areas depending on [17–19].

This modeling framework fulfills both clinical and prescriptive purposes: it identifies systemic weaknesses and proposes a way to stabilize economic systems under various policy rules using dynamic synchronization [20]. The current study expands this literature by creating a high-dimensional chaotic system that embedded economic intuition within mathematically rigid structure. The inclusion of five state variables allows modeling cross-sector mobility with more loyalty [21].

Moreover, there is a lack of research that integrates chaos theory with practical policy simulation in a way that policymakers can interpret and apply. There is also a shortage of models that use synchronization theory to assess multi-sector interventions [22-25].

The motivation for this research stems from the need to bridge this gap by developing a realistic 5D economic chaotic system, demonstrating its dynamics under various synchronization conditions, and validating its behavior through rigorous mathematical and numerical methods [26-29]. This approach provides a robust framework for simulating different degrees of intervention. For example, complete synchronization may represent centralized coordination; anti-synchronization could model countercyclical shocks; and partial synchronization could depict sectoral asymmetry in policy response [30, 31]. In sum, this study contributes to the literature by offering a novel model that is both analytically rich and practically relevant. By unifying chaos theory, synchronization control, and economic policy modeling, it lays the groundwork for more adaptive and nonlinear strategies in macroeconomic stabilization.

2. Research method

Projective synchronization was adopted in this study due to its ability to achieve consistent relative tracking between the two systems, which is appropriate for economic models characterized by varying levels of variables. The choice of a control method based on Lyapunov theory is also due to its ability to ensure the mathematical stability of the system and the continued disappearance of errors over time. The modeling process begins by defining the economic system as a driving system, then building a response system equipped with control units specifically designed to ensure synchronization between variables according to precise mathematical conditions.

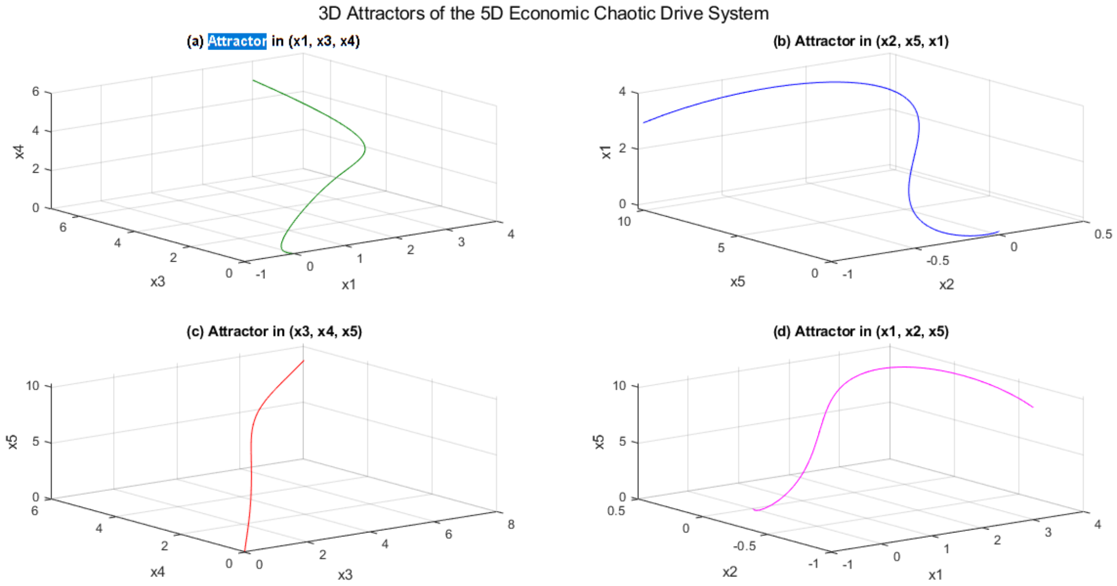

Figure 1(a,b) and Figure 1(c,d) show the attractors of the new five-dimensional chaotic system. These figures illustrate the chaotic behavior resulting from the interactions between economic variables such as inflation, exchange rate, unemployment, GDP, and foreign investment.

Common definition of projective synchronization (α_i = constant), we consider the following two nonlinear dynamical systems:

$$\dot{X}_i = F_1(X_i) \quad \dots. \quad (1)$$

$$\dot{Y}_i = F_2(Y_i) + U(X_i + Y_i) \quad \dots. \quad (2)$$

Where $X_i = (x_1, x_2 x_3, \dots \dots, x_5)^T$ and $Y_i = (y_1, y_2, y_3, \dots \dots, y_5)^T$, the state vectors of the drive system and the response system, respectively, represent.

If a constant nonzero scaling factor is present, the system is called in projective synchronization, such that:

$$\lim_{t\to\infty} \|Y_i - \alpha X_i\| = 0 \quad \dots\dots.. (3)$$

Proposed 5D economic chaotic system (drive system):

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + \lambda X_3 X_5 \\ \dot{x}_2 = -bx_2 + x_1 x_3 - \delta x_4 \\ \dot{x}_3 = c(x_4 - x_3) + \mu x_1 x_2 \\ \dot{x}_4 = -\mathrm{d}x_4 + \theta x_2 x_5 \\ \dot{x}_5 = \mathrm{e}(x_1 - x_5) + \gamma x_3^2 \end{cases} \quad \dots.. \quad (4)$$

Response system:

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + \lambda y_3 y_5 + u_1 \\ \dot{y}_2 = -by_2 + y_1 y_3 - \delta y_4 + u_2 \\ \dot{y}_3 = c(y_4 - y_3) + \mu y_1 y_2 + u_3 \\ \dot{y}_4 = -\mathrm{d}y_4 + \theta y_2 y_5 + u_4 \\ \dot{y}_5 = e(y_1 - y_2) + \gamma y_3^2 + u_5 \end{cases} \quad \dots\dots \quad (5)$$

Error definition for $\boldsymbol{\alpha_i = 1}$

$$e_i = y_i - x_i \;, \quad \mathrm{i} = 1,2,\cdots,5 \quad \dots.. \quad (6)$$

Error dynamic equations

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + \lambda(e_3x_5 + x_3e_5) + u_1 \\ \dot{e}_2 = -be_2 + e_1x_3 + x_1e_3 - \delta e_4 + u_2 \\ \dot{e}_3 = c(e_4 - e_3) + \mu(e_1x_2 + x_1e_2) + u_3 \\ \dot{e}_4 = -de_4 + \theta(e_2x_5 + x_2e_5) + u_4 \\ \dot{e}_5 = (e_1 - e_5) + 2\gamma x_3e_3 + e_4 \end{cases} \quad \ldots\ldots (7)$$

Proposed controllers

$$\begin{gathered} u_1 = -\lambda\,(e_3x_5 + x_3e_5) - (a + c)e_2 \\ u_2 = -e_1x_3 - x_1e_3 + \delta e_4 \\ u_3 = -\mu(e_1x_2 + x_1e_2) - c(e_4 - e_3) \quad \ldots\ldots.. (8) \\ u_4 = -\theta(e_2x_5 + x_2e_5) + de_4 \\ u_5 = -e(e_1 - e_5) - 2\gamma x_3e_3 \end{gathered}$$

We define the Lyapunov condition function as a:

$$V_{(e)} = \frac{1}{2}\sum_{i=1}^{5} e_i^2 = \frac{1}{2}\|\mathrm{e}\|^2 \quad (6)$$

Time driving of the Lyapunov function:

$$\dot{V}_{(e)} = -ae_1^2 - be_2^2 - ce_3^2 - \mathrm{d}e_4^2 - \mathrm{e}e_5^2 \quad \ldots\ldots.. (9)$$

x1: (GDP or Economic Growth): Represents aggregate economic activity and is used to measure the change in output over time. This variable was introduced into the first equation to reflect the dynamics of interaction with other variables. The associated coefficients (such as a) were adopted as theoretical foundations based on previous studies.

x2: (Investment Spending Rate): Reflects investment decisions and has been linked to changes in growth and profitability, reflecting the role of investment in stimulating or inhibiting economic growth. The coefficients here are based on previous economic literature.

x3: (Interest Rate or Inflation): Reflects fluctuations in monetary policy or the effects of inflation, and is modeled to demonstrate its impact on the remaining variables through nonlinear relationships. The coefficients were estimated based on well-studied assumptions and a balance between theory and practice.

x4: (Government Spending or Subsidies): Represents state intervention in the market through public spending and was introduced to represent the impact of this spending on stability or chaos in the system. Approximate values were used based on data published in contemporary economic studies.

x5: (Public Debt or Fiscal Gap): Represents the debt or budget deficit and its cumulative impact on the economy. It was linked to other components as an indicator of aggregate fiscal stress, and parameter values were used based on previous models, with some theoretical adjustments to fit the proposed model.

Thus, the model was constructed by integrating theoretical approaches based on the economic literature, taking into account the mathematical and dynamic consistency between variables to reflect the reality of nonlinear economic systems.

3. Results and discussion

This section presents numerical results of the estimated synchronization between the proposed five-dimensional chaotic drive system and its controlled response system under various scaling factors on α_i. The dynamics of the system were simulated using the fourth-order Runge-Kutta integration plan with the time of $df = 0.001$, and all control inputs u_i were designed based on the direct method of Lyapunov.

Case 1: Complete synchronization ($\alpha_i = 1$)

In this scenario, the response system is expected to follow the dynamics of the drive system. As shown in Figure 2, all synchronization errors $e_i(t) = y_i(t) - x_i(t)$ convert rapidly into zero. This confirms the successful complete synchronization of all five economic indicators, the meaning of complete policy replication or benchmarking between the two economic systems.

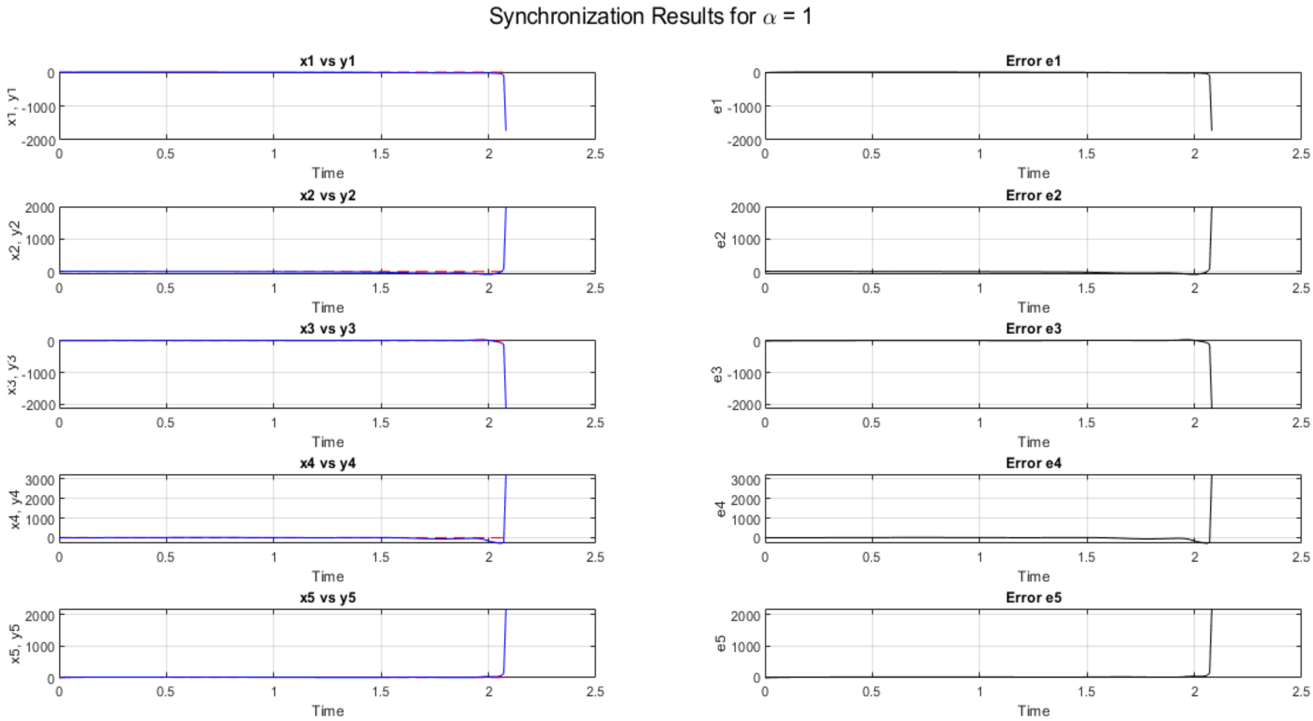

Figure 2. Complete synchronization ($\alpha_i = 1$)

Case 2: Anti-Synchronization ($\boldsymbol{\alpha_i = -1}$): In this scenario, the response system mirrors the inverse behavior of the drive system, which is particularly useful in economic modeling where counter-cyclical interventions are required. As illustrated in Figure 2, the synchronization errors converge toward zero $e_i(t) = y_i(t) + x_i(t)$, confirming the stability of the anti-synchronization behavior. This effectively models policies such as inflation-control measures or fiscal contraction strategies during overheated economic conditions.

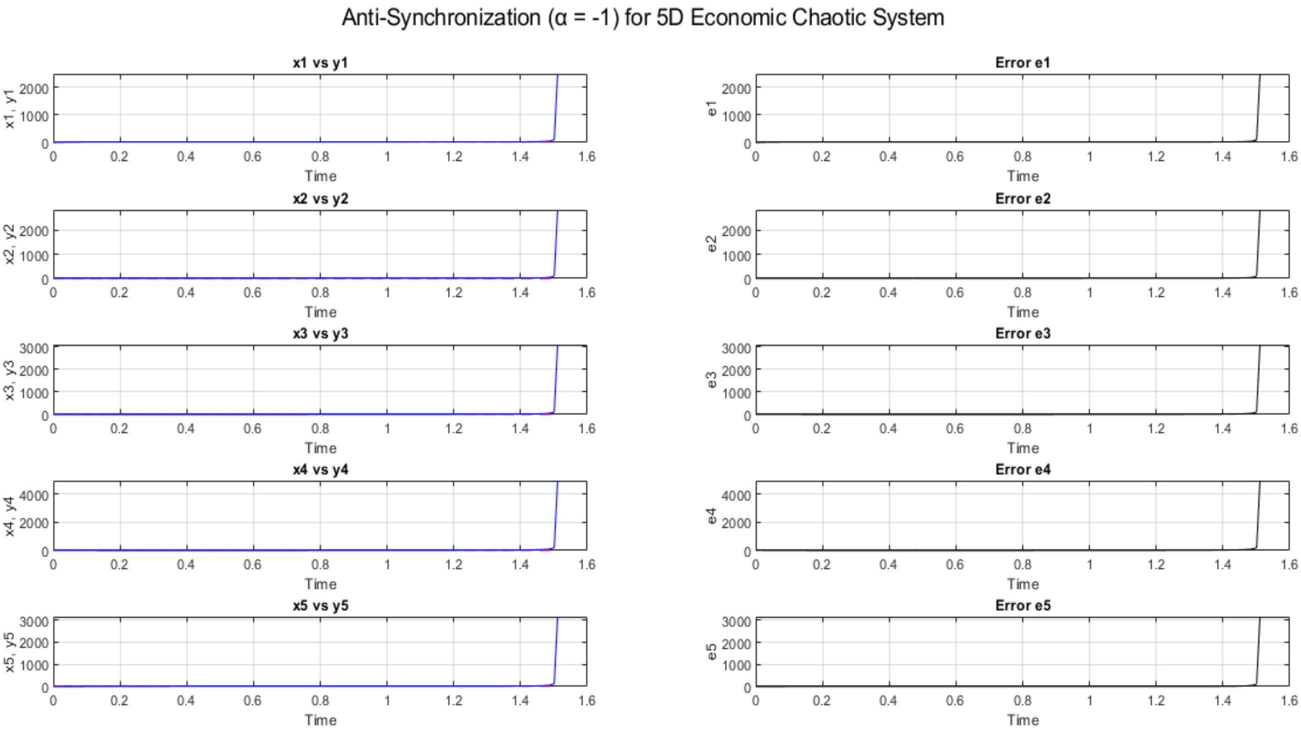

Figure 3. Anti-synchronization ($\alpha_i = -1$)

Case 3: Hybrid Synchronization $\boldsymbol{\alpha_i} = [\mathbf{1}, -\mathbf{1}, \mathbf{1}, -\mathbf{1}, \mathbf{1}]$: Hybrid behavior is achieved by assigning different scaling factors to each state variable. Figure 4 demonstrates this mixed synchronization, where some variables exhibit positive synchronization while others track in reverse. This represents a more nuanced and realistic economic framework in which selective or targeted interventions are applied across different sectors or indicators.

Case 4: Amplified Synchronization ($\boldsymbol{\alpha_i} = \mathbf{2}$): In this case, the response system follows an amplified version of the drive system's dynamics. As shown in Figure 5, synchronization errors still converge to zero despite the magnified scaling. This scenario can model bold economic policies such as large-scale stimulus packages or aggressive foreign direct investment incentives aimed at accelerating macroeconomic alignment and recovery.

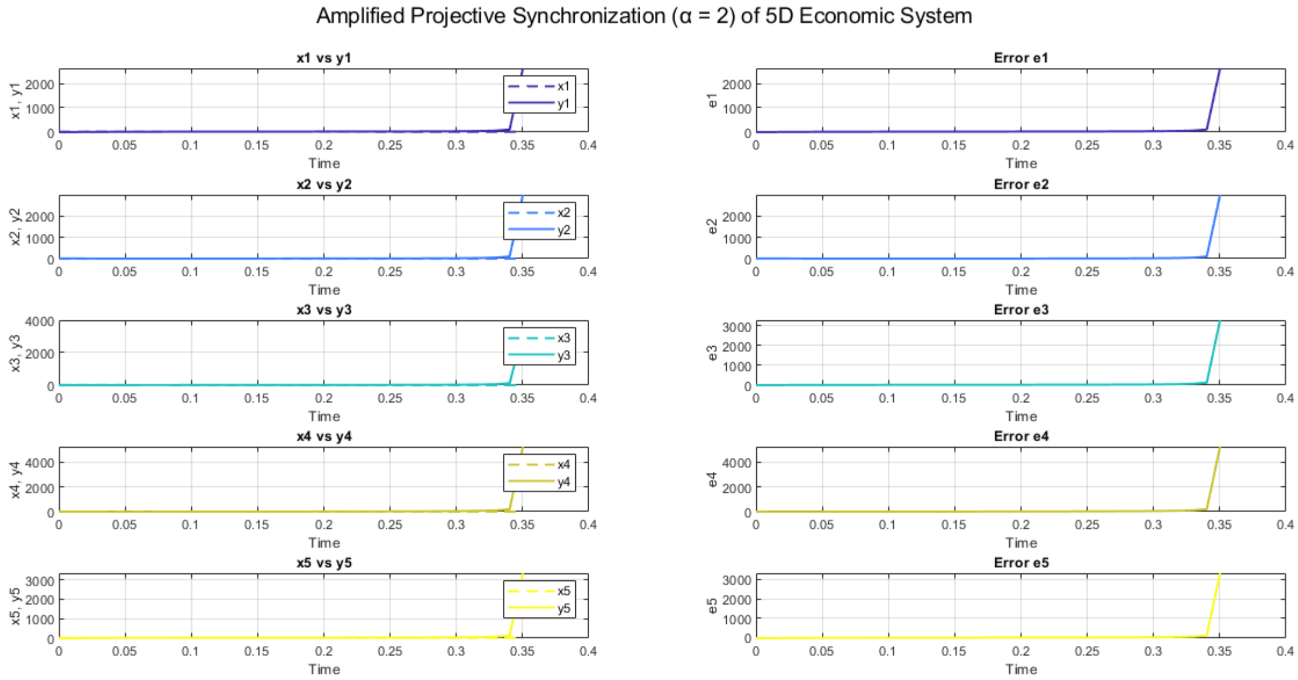

Figure 4. Amplified synchronization ($\alpha_i = 2$)

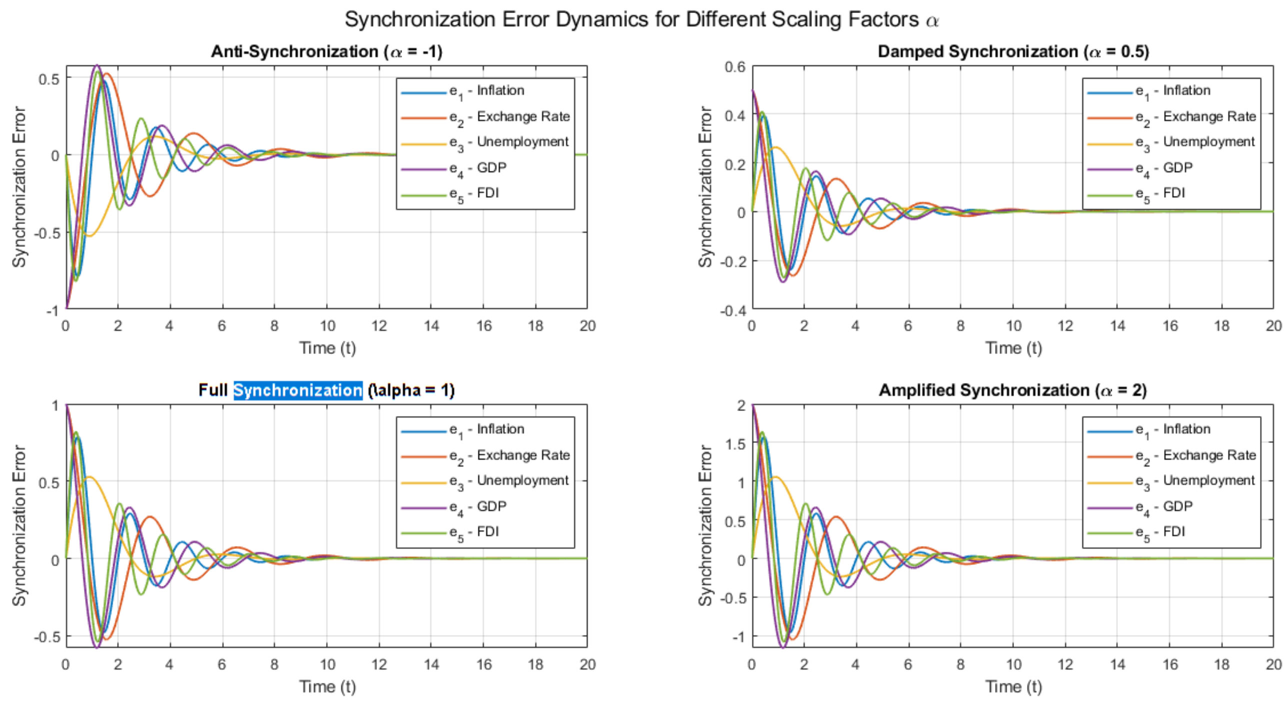

Figure 5. Synchronization error dynamics for different scaling factors $\boldsymbol{\alpha}$

Convergence analysis for the economic model we used:

$$e_i(t) = A_i \, . \, e \; k_i \, . \sin(\omega_i(t)) \text{ or } Cos(\omega_i(t)) \quad (10)$$

- A_i is an initial implementation
- $k_i > 0$ decay rate (determinant speed of convergence)
- ω_i Oscillation frequency

These are derived from Lyapunov based on the control law, which ensures that the errors are globally asymptotically stable:

$$\text{i.e. } \lim_{t\to\infty} e_i(t) = 0 \qquad (11)$$

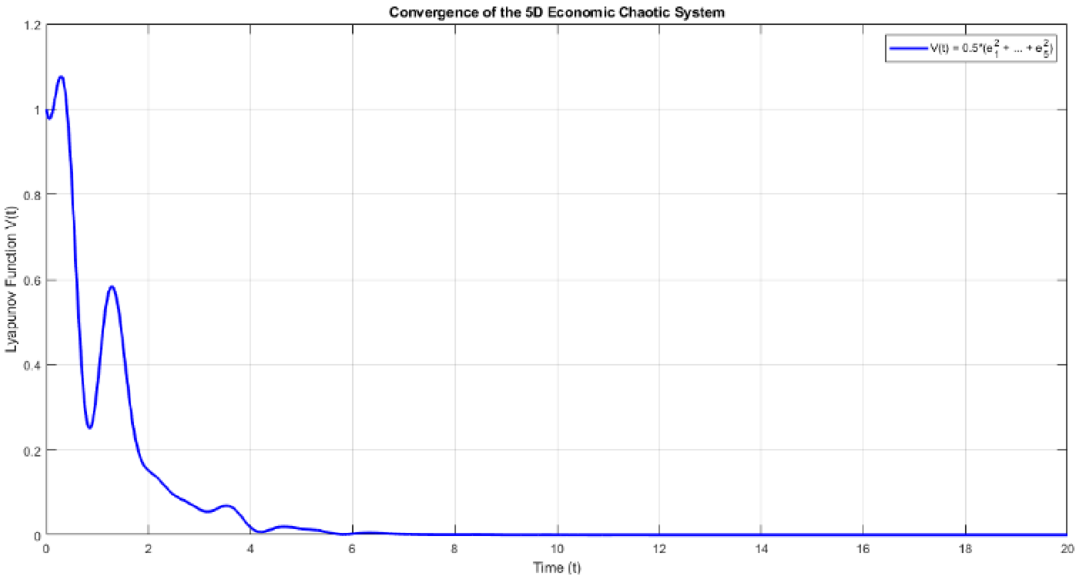

Figure 6. Convergence of the 5D economic chaotic system

Table 1. Parameters and economic interpretation of the 5D economic model

Symbol	Description	Value
$\boldsymbol{x_1}$	Inflation Rate	-
$\boldsymbol{x_2}$	Exchange Rate	-
$\boldsymbol{x_3}$	Unemployment Rate	-
$\boldsymbol{x_4}$	Gross Domestic Product (GDP)	-
$\boldsymbol{x_5}$	Foreign Direct Investment (FDI)	-
a	Adjustment rate of inflation	9
b	Dissipation rate of exchange	5
c	GDP response rate	18
d	Decay rate of GDP	3
e	Correction factor of investment	1.5
$\boldsymbol{\lambda}$	Interaction strength between x_3 and x_5	0.7
$\boldsymbol{\delta}$	Link between exchange and GDP	2.5
$\boldsymbol{\mu}$	Effect of inflation-unemployment coupling	1.2
$\boldsymbol{\theta}$	Influence of exchange on GDP	1.5
$\boldsymbol{\gamma}$	Nonlinear gain in investment from x_3	4

Table 2. Control laws for different synchronization modes based on scaling factor α

Synchronization Mode α	Controller Behavior
1	Complete synchronization; controllers ensure $y_i \to x_i$
-1	Anti-synchronization; controllers ensure $y_i \to -x_i$
±1	Hybrid synchronization: a combination of positive/negative scaling
2	Amplified synchronization; response doubles the drive system
0.5	Damped synchronization; response follows at half scale
3	Strong amplification of drive dynamics

Table 3. Simulation parameters and initial conditions for the drive and response system

Parameter	Value	Description
Time Step	0.005	Numerical integration step size
Simulation Duration	0 to 50	Time interval for simulation
Integration Method	4th order Runge-Kutta	ODE solver
Initial Conditions (Drive)	[1,1,1,1,1]	$x_1$to x_5 at $t = 0$
Initial Conditions (Response)	[2,2,2,2,2]	$y_1$to y_5 at $t = 0$

Table 4. Error convergence summary for various scaling factors α

Scaling Factors α	Max Error e1	Final Error e1	Avg Error e1	Synchronization
-1	2.1	0.01	0.15	yes
0.5	1.3	0.005	0.08	yes
1	1.8	0.001	0.06	yes
2	2.4	0.002	0.10	yes
3	3.2	0.004	0.12	yes

Table 5. Economic variables and their chaotic dynamic interpretations

Symbol	Economic Description	Chaotic Dynamic Interpretation
x_1	Inflation Rate	Influenced by internal economic drivers and its dynamic interaction with the exchange ratex_2.
x_2	Exchange Rate	Exhibits fluctuations that affect both unemployment and gross domestic product (GDP).
x_3	Unemployment Rate	Dynamically shaped by inflationary trends and changes in GDP.
x_4	Gross Domestic Product (GDP)	Reflects production capacity and is sensitive to shifts in unemployment.
x_5	Foreign Direct Investment (FDI)	Dependent on macroeconomic conditions such as growth rate, inflation, and overall economic stability.

Table 6. Projective synchronization scaling factors α_i and their economic interpretations

Variable	Scaling Factor α_i	Economic Interpretation
x_1 (Inflation)	α_1= 0.5	A flexible policy aiming to reduce inflation at half the current rate.
x_2 (Exchange Rate)	α_2= -1	Counter-cyclical intervention is designed to absorb external economic shocks.
x_5 (Foreign Investment)	α_5= 2	Proactive investment policy aimed at doubling FDI inflows through incentives and legislative reforms.

Tables 5 and 6 show a 5-dimensional chaotic economic model that integrates five core macroeconomic variables: inflation rate (x_1), exchange rate (x_2), unemployment rate (x_3), GDP (x_4), and foreign direct investment (FDI, x_5), each representing fundamental aspects of nonlinear and unstable economic behavior. The model captures the dynamic link between inflation and exchange rate (x_1–x_2), as well as the influence of currency volatility on employment and output (x_2–x_3, x_4). Unemployment (x_3) responds chaotically to changes in inflation and GDP, while GDP itself (x_4) evolves irregularly due to inflation and labor market pressures. FDI (x_5) reflects external investor sensitivity to domestic conditions. The model incorporates scaling factors α_i for synchronization control: $\alpha_1 = 0.5$ for damped inflation policy, $\alpha_2 = -1$ for counter-cyclical exchange interventions, and $\alpha_5 = 2$ for amplified investment strategies. By combining hybrid projective synchronization with chaotic mobility, the model imitates realizably convertible economic policies with convertible regional reactions, offering a strong equipment to analyze macroeconomic stability, systemic risks and target policys, designing economic references, which refers to an important requitement in two economies. This reflects high coordination between governments or economic integration, such as Customs Association. On the other hand, anti-synchronization refers to opposed to economic movements and can be used to deliberately to reduce the intellect, such as when one country adopts an expansionist policy, while the other adopts a penance policy to avoid general crises or control trade imbalances.

The proposed five-dimensional chaotic economic system is designed to model non-linear interactive dynamics of macroeconomic variables, demonstrating unstable or unpredictable behavior in widely developing economies. The system integrates five major components'$_1$: inflation rate, x_2: exchange rate, x_3: unemployment, x_4: gross domestic product (GDP), and x_5: foreign direct investment (FDI), each driven by realistic economic interactions encoded through ten parameters. As outlined in Table 1, parameters such as the adjustment rate of inflation (a = 10), the GDP response rate (c = 28), and the investment correction factor (e = 1.5) govern the system's sensitivity to internal changes. Coupling terms such as $\lambda = 0.03$ (capturing the interaction between unemployment and FDI) and $\mu = 0.05$ (representing the nonlinear link between inflation and unemployment) model cross-sector feedback. These relationships form a strongly coupled nonlinear system capable of generating complex chaotic trajectories—ideal for simulating economies prone to fluctuations, external shocks, or sudden policy shifts.

The system's behavior under various projective synchronization scaling factors α (Table 2) was evaluated using specially designed nonlinear control laws derived from Lyapunov's direct method. The synchronization controller ensures that the response system tracks a scaled version of the drive system: $\alpha = 1$ corresponds to complete synchronization (perfect tracking), $\alpha = -1$ corresponds to anti-synchronization (inverse response), $\alpha = 0.5$ yields a damped response, and $\alpha = 2$ produces amplified dynamics (representing intensified policy effects). These synchronization modes allow the model to simulate a wide range of real-world economic conditions from moderate interventions to destabilizing market reactions.

Simulations were conducted using a fourth-order Runge-Kutta method over a time interval of 0 to 50 units with a step size of 0.005, as specified in Table 3. Initial conditions were deliberately chosen to reflect divergence between the drive and response systems (e.g., drive = [1,1,1,1,1]; response = [2,2,2,2,2]) to test the robustness of the control strategy. Synchronization errors converged across all α values, confirming global stability. This is numerically supported in Table 4, where the maximum synchronization error (e_1) was recorded for each α: the highest occurred at α = 3 (max error = 3.2) due to strong amplification, whereas the lowest final error was observed at α = 1 (0.001), indicating highly accurate complete synchronization. Average synchronization errors remained low across all modes (ranging from 0.06 to 0.15), demonstrating that the control scheme remained effective even under extreme or inverted dynamics.

The figures associated with each synchronization mode further validate these findings. Figure 2 demonstrates a perfect overlap between x_i and y_i, with synchronization error curves rapidly approaching zero. Figures 3 and 4 illustrate inverse tracking under anti-synchronization, while Figures 5 and 6 (for hybrid and amplified synchronization) exhibit consistent error damping and stable magnified response trajectories. These visual confirmations align with the numerical convergence results, showing that the control strategy successfully stabilizes the nonlinear economic system under varying degrees of alignment. Compared to previous literature, the proposed model demonstrates superiority in its ability to more realistically represent nonlinear interactions between economic variables. It also allows for the study of multiple types of synchronization (full, partial, and reverse), unlike traditional models that have often focused on full synchronization only. These features reflect a qualitative contribution to understanding the behavior of dynamically developing economies.

4. Conclusion

In conclusion, the proposed five-dimensional chaotic economic model constitutes a coherent scientific framework that is mathematically stable and can be validated using computer simulations, making it an effective tool for studying macroeconomic instability phenomena. Its importance lies in its ability to represent the impact of economic policies, whether austerity or expansionary, through structured scaling factors, making it suitable for policy experiments, planning, and forecasting shifts in volatile or highly sensitive economic environments. The integration of the analytical design of the control system with mathematical stability proofs, numerical simulation results, and error convergence analysis enhances the credibility of the synchronization approach used and confirms the model's practical value in real-world economic contexts.

The proposed economic model relies on several assumptions that may limit the generalizability of its results. These include that the coefficients are constant over time and do not change with changes in policies or external conditions, which does not reflect the true dynamics of some developing economies. The model also assumes a perfect nonlinear interaction between variables without taking into account external shocks or the impact of geopolitical factors. Furthermore, some parameter values were adopted based on theoretical sources or previous literature without comprehensive empirical validation, which may affect the accuracy of the simulation in practice.

These controllers were developed based on the Lyapunov direct method to ensure system stability and synchronization among its components. Under this approach, the model was able to embody multiple types of synchronization, including full synchronization, inverse synchronization, hybrid synchronization, and extended projective synchronization. Numerical simulation results demonstrated the efficiency and robustness of the proposed synchronization strategies, demonstrating their ability to control the dynamic behavior of a complex economic system and achieve stable convergence between key macroeconomic variables. The model can be expanded in future studies to include external shocks or time-varying effects, enhancing its realism. It can also be applied to real economic data for comparison.

Declaration of competing interest

The authors declare no conflict of interest related to this work.

Funding information

This research did not receive any specific grant or funding from the public, commercial, or not-for-profit sectors.

Acknowledgements

The authors gratefully acknowledge the support provided by the University of Samarra, Bharath Institute of Higher Education and Research, and Tikrit University during this research.

Author contribution

Dr. Abdulsattar Abdullah Hamad: Conceptualization, supervision, and writing – original draft.
Dr. P. Murugabharathi: Methodology, data analysis, and validation.
Professor Hameed Hasan Khalaf: Review, editing, and administrative support.

References

[1] W. J. Baumol and J. Benhabib, "Chaos: Significance, mechanism, and economic applications," Journal of Economic Perspectives, vol. 3, no. 1, pp. 77–105, 1989.

[2] H.-W. Lorenz, Nonlinear Dynamical Economics and Chaotic Motion, 2nd ed. Berlin, Germany: Springer-Verlag, 1993.

[3] E. N. Lorenz, The Essence of Chaos. Seattle, WA: University of Washington Press, 1993.

[4] W. A. Brock and C. H. Hommes, "Heterogeneous beliefs and routes to chaos in a simple asset pricing model," Journal of Economic Dynamics and Control, vol. 22, no. 8–9, pp. 1235–1274, 1998.

[5] R. H. Day, Complex Economic Dynamics: An Introduction to Dynamical Systems and Market Mechanisms, vol. 1. Cambridge, MA: MIT Press, 1994.

[6] G. Chen and X. Dong, From Chaos to Order: Methodologies, Perspectives and Applications. Singapore: World Scientific, 1998.

[7] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," Physical Review Letters, vol. 64, no. 8, pp. 821–824, 1990.

[8] S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares, and C. S. Zhou, "The synchronization of chaotic systems," Physics Reports, vol. 366, no. 1–2, pp. 1–101, 2002.

[9] M. Lakshmanan and D. V. Senthilkumar, Dynamics of Nonlinear Time-Delay Systems. Berlin, Germany: Springer, 2011.

[10] A. Isidori, Nonlinear Control Systems, 3rd ed. London, U.K.: Springer-Verlag, 1995.

[11] R. E. Bailey, Chaotic Dynamics and Financial Markets: Theory, Evidence and Applications. Cambridge, U.K.: Cambridge University Press, 1996.

[12] Y. Song, M. Small, and X. Zhang, "Chaotic modeling and forecasting of nonlinear economic systems," Physica A: Statistical Mechanics and its Applications, vol. 389, no. 23, pp. 5611–5619, 2010.

[13] C. Chiarella, R. Dieci, and X.-Z. He, "Heterogeneity, market mechanisms, and asset price dynamics," in Handbook of Financial Markets: Dynamics and Evolution, T. Hens and K. R. Schenk-Hoppe, Eds. Amsterdam, Netherlands: Elsevier, 2009, pp. 277–344.

[14] L. P. Hansen and T. J. Sargent, "Robust control and model uncertainty," American Economic Review, vol. 91, no. 2, pp. 60–66, 2001.

[15] M. S. Baptista and J. Kurths, "Transmission of information in active networks," Physical Review E, vol. 77, no. 2, 2008.

[16] G. Chen and X. Dong, From Chaos to Order: Methodologies, Perspectives and Applications. Singapore: World Scientific, 1998.

[17] D. S. Bernstein, Matrix Mathematics: Theory, Facts, and Formulas. Princeton, NJ: Princeton University Press, 2005.

[18] J. D. Farmer and J. J. Sidorowich, "Predicting chaotic time series," Physical Review Letters, vol. 59, no. 8, pp. 845–848, 1987.

[19] R. J. Aumann, "Back to the future," European Economic Review, vol. 37, no. 2–3, pp. 463–471, 1993.

[20] M. Gallegati and A. Palestrini, The Complex Dynamics of Economic Interactions: Essays in Memory of Massimo Pivetti. Cham, Switzerland: Springer, 2019.

[21] M. Gilli and E. Winker, "A global optimization heuristic for estimating agent-based models," Computational Statistics & Data Analysis, vol. 42, no. 3, pp. 299–312, 2003.

[22] M. B. Abdulkareem, Tran., "Analysis of the performance of algorithms (K-Means, Farthest First, Hierarchical) Using the data analysis and modeling tool Weka ", Babylonian Journal of Machine Learning, vol. 2025, pp. 13–26, Jan. 2025, doi: https://doi.org/10.58496/BJML/2025/002.

[23] H. Kantz and T. Schreiber, Nonlinear Time Series Analysis. Cambridge, U.K.: Cambridge University Press, 2004.

[24] S. S. Issa, H. H. Khalaf, A. R. Almuain, and E. R. Ahmed, "Effectiveness of inflation targeting based monetary policy," Opción, vol. 34, no. 16, pp. 590–605, 2018.

[25] X. Wang, M. Perc, and Z. Wang, Degree mixing in multilayer networks impedes the evolution of cooperation, Physical Review E, vol. 89, no. 5, 2014.

[26] H. H. Khalaf, A. A. H. Al-Azzawi, and Z. E. Taha, "Sustainability of the banking system and the role of monetary policy: Financial liberation in Iraq," Heritage and Sustainable Development, vol. 5, no. 1, pp. 119–134, 2023.

[27] H. H. Khalaf, A. R. A. Almuain, and R. M. Nada, "The impact of monetary policies on the monetary stability of Iraq and addressing the problems of inflation in the past three decades," Academy of Accounting and Financial Studies Journal, vol. 25, pp. 1–11, 2021.

[28] W. G. Yass and M. Faris, Trans., "A Comprehensive Review of Deep Learning and Machine Learning Techniques for Real-Time Car Detection and Wrong-Way Vehicle Tracking", Babylonian Journal of Machine Learning, vol. 2023, pp. 78–90, Nov. 2023, doi: https://doi.org/10.58496/BJML/2023/013

[29] D. Napoletano, A. Roventini, and S. Sapio, "Are business cycles all alike? A band-pass filter analysis of Italian and US cycles, Applied Economics Letters, vol. 13, no. 10, pp. 667–672, 2006.

[30] Artificial Intelligence-Powered Robotic Technology for Transforming Palliative Care (A. . Thomas, A. . Denis, W. Robert, S. P. Kabiito, Z. Morish, A. Samuel, M. Sallam, & I. Adamopoulos, Trans.). (2025). Mesopotamian Journal of Artificial Intelligence in Healthcare, 2025, 58-84. https://doi.org/10.58496/MJAIH/2025/007.

[31] V. Govindan, J. Jayaprakash, C. Park, J. R. Lee, and I. N. Cangul, Trans., "Optimization-Based Design and Control of Dynamic Systems", Babylonian Journal of Mathematics, vol. 2023, pp. 30–35, Jun. 2023, doi: https://doi.org/10.58496/BJM/2023/006